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PROGRESS REPORT ON

Geostroptic Vortex Motion II. Application to Hurricane Tracking

G. K. MORIKAWA

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Progress Report on

GEOSTROPHIC VORTEX MOTION II.
APPLICATION TO HURRICANE TRACKING

G. K. Morikawa

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1. SUMMARY AND INTRODUCTION

In IMM-NYU Report[1] No. 226 (1956), the equations governing geostrophic vortex motion were derived and the possibility of applying the equations to the problem of hurricane tracking was discussed. Since that time E. Isaacson, D. A. Levine and the author have had some success in the short range (one or two days) prediction of hurricane motion over the Atlantic Ocean approaching the eastern seaboard. In this report we present an outline of the work and, briefly, what is planned for the immediate future.

The basic equations of geostrophic vortex motion derived in GVM are

$$\frac{d^{(1)}}{d\tau}(\Delta-\kappa^2)\psi = 0 \tag{1a}$$

where

$$\frac{d^{(1)}}{d\tau}(\quad) = (\quad)_\tau+u^{(1)}\cdot(\quad)_x+v^{(1)}\cdot(\quad)_y \tag{1b}$$

and

$$u^{(1)} = -\psi_y\ ,\quad v^{(1)} = \psi_x\ . \tag{1c}$$

The third order non-linear differential equation (1a) for the stream function $\psi \equiv gh^{(1)}/f$ is called the Geostrophic Conservation Equation. Subscripts denote partial differentiation; $\Delta(\quad) = (\quad)_{xx}+(\quad)_{yy}$ is the two dimensional Laplacian operator;

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[1] Based on this report, a manuscript titled "Geostrophic Vortex Motion" has been submitted to the Journal of Fluid Mechanics, Cambridge University, England, for publication; in this report we refer to this paper as GVM.

$\kappa^2 = f^2/gh_o$ where $f(=$ constant) is the Coriolis parameter, g is the gravitational acceleration and h_o is the depth of the homogeneous atmosphere at rest; and $\quad = \alpha t$ is the scaled time where $\alpha \ll 1$ is a small parameter related to the strength of disturbances. The superscript (1) denotes that the solution $(h^{(1)}, u^{(1)}, v^{(1)})$ is the first order approximation to the exact solution $(h,u,v) = (h_o + \alpha h^{(1)} + \ldots, \alpha u^{(1)} + \ldots, \alpha v^{(1)} + \ldots)$ expressed as a power series expansion with respect to the parameter α, *perturbed on the flow at rest* $(h,u,v) = (h_o,0,0)$.

The singular vortex solution of (1) which vanishes at infinity is

$$\psi_o(|r-r_o|) = -\frac{\gamma}{2\pi} K_o(\kappa|r-r_o|) \,. \tag{2a}$$

The vortex strength γ is defined by

$$\gamma = \oint_C \psi_\nu ds - \kappa^2 \iint_R \psi dxdy \tag{2b}$$

where the integration is over a region R with boundary C which encloses the vortex and ν denotes differentiation in the outward normal direction to C. K_o is the modified Bessel function of the second kind and zero order of argument $\kappa|r-r_o|$ where

$$|r-r_o|^2 = [x-x_o(\tau)]^2 + [y-y_o(\tau)]^2 \tag{2c}$$

(2a) is called the *Geostrophic Point Vortex*.

For application of equations (1) and (2) to the problem of hurricane tracking we write the stream function ψ as the sum of the singular solution (2a), representing the hurricane, and the remaining continuous regular solution ψ_1 (see GVM):

$$\psi = \psi_o(|r-r_o|)+\psi_1(x,y,\tau) \quad . \tag{3a}$$

Putting (3a) in (1) we obtain the equations of motion of a single geostrophic point vortex embedded in a continuous flow field (see GVM):

$$\frac{d^{(1)}}{d\tau}(\Delta-\kappa^2)\psi_1 = 0 \tag{3b}$$

where

$$u^{(1)} = -(\psi_o+\psi_1)_y \ , \quad v^{(1)} = (\psi_o+\psi_1)_x \tag{3c}$$

and

$$\frac{dx_o}{d\tau} = -(\psi_1)_y \tag{3d}$$

$$\frac{dy_o}{d\tau} = (\psi_1)_x \tag{3e}$$

where $-(\psi_1)_y$ and $(\psi_1)_x$ are the x- and y-components of the velocity of the continuous flow field ψ_1 evaluated at the vortex point (x_o,y_o).

When the initial and boundary conditions are given, either the system of equations (1) or (3) can be applied to the problem of hurricane tracking.[2] There is some feeling among meteorologists that, in numerical prediction of hurricane motion, the hurricane must be explicitly distinguished from the rest of the flow as shown in (3a). Otherwise the hurricane which is a strongly concentrated disturbance might not be resolvable by the usual spacial grid spacing (one to two hundred miles). Our investigations so far indicate that equations (1) can be directly

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[2] In this report we call the application of equations (1) the Total Flow Method, and the application of equations (3) the Point Vortex Method.

applied without losing the hurricane in the grid mesh. Thus, our present position on this question is that advantages and disadvantages exist for either method of hurricane tracking and an important part of our overall program is to determine the conditions under which one method is, if at all, better than the other. It is rather apparent that using equations (1) entails less numerical work. Also, the determination of preference of one method over the other appears to be strongly dependent on the reliability and accuracy of the initial data particularly in the neighborhood of the eye of the hurricane, i.e., of the order of one hundred miles radius. Thus we expect some shift of the relative advantages of one method over the other as better initial data is obtained.

The remainder of this report is divided into three sections. In §2 we summarize the successful test run on the NYU UNIVAC of a known solution of equations (3). The results have been stated in a previous report IMM-NYU 247 (1958) by D. A. Levine in which the numerical method of solution of the Geostrophic Conservation Equation (1) is described. In §3 we describe the first series of 24 hour runs (hourly time step) by the Total Flow Method, equations (1), using initial data for Hurricane Ione, 1955. Although the boundary data was held fixed in this test series in order to simplify the machine coding, a partially successful prediction was made. In §4 some partial runs by the Point Vortex Method, equations (3), are described and some of the difficulties encountered are discussed.

2. NUMERICAL SOLUTION OF AN EXACT SOLUTION

The machine code described in IMM-NYU 247 was tested against an exact solution of the geostrophic conservation equation (1). This test served four main purposes: 1) to test the functioning of the code, 2) to verify estimates of error of the numerical scheme, 3) to establish the validity of the exact singular solution and 4) to test the proper scaling of physical quantities.

The exact solution used for the test is

$$\psi = \psi_o + \psi_1 = -\frac{\gamma}{2\pi} K_o(\kappa|r-r_o|) + Ae^{-\kappa y} \tag{4a}$$

which satisfies (1). $\psi_1 = Ae^{-\kappa y}$ is time-independent. The motion of the vortex by (3d,e) is

$$\frac{dx_o}{d\tau} = -\ Ae^{-\ y_o} \tag{4b}$$

$$\frac{dy_o}{d\tau} = 0 \tag{4c}$$

where we have put the geostrophic point vortex initially at the origin. (4b,c) states that the vortex simply moves uniformly from west to east:

$$x_o(\tau) = A \tag{4d}$$

$$y_o(\tau) = 0 . \tag{4e}$$

In the numerical solution, the singular $K_o(\kappa|r-r_o|)$ was truncated so that the maximum tangential velocity was 120 mph:

$$[(\psi_o)_r]_{max} = \frac{\gamma\kappa}{2\pi}K_1(\kappa r_c) \ . \qquad (5)$$

The constants chosen were $\gamma/2\pi = 10^4 \text{miles}^2\text{hr}^{-1}$ and $\kappa = 3.65 \times 10^{-4}$ miles^{-1}; then the truncation or cut-off radius $r_c = 82$ miles. The constant A was chosen so that the translational velocity of the point vortex $\kappa A = 20$ mph. 24 hourly time steps and a mesh width of 200 miles was used for a 20 by 20 grid. The test results were remarkably good:

1) The point vortex moved due east at exactly 20 mph (to six significant figures).

2) The regular flow field ψ_1 remained time-independent (to six significant figures).

The exact solution (4a) is a special case of a more general solution where the regular field ψ_1 is

$$\psi_1(x,y,\tau) = \int_0^{2\pi} F(\theta,\tau)\exp\left\{\pm\kappa(x\cos\theta + y\sin\theta)\right\}d\theta + \text{constant} \ , \qquad (6)$$

with $F(\theta,\tau)$ an arbitrary function. (6) may be useful for additional computational tests.

3. NUMERICAL SOLUTION BY THE TOTAL FLOW METHOD

In the total flow method we do not distinguish between the hurricane and the rest of the flow field but introduce the given initial and boundary data directly into (1). It is conceivable that if the hurricane is too localized then it will not be resolvable by the grid mesh size used (200 nautical miles). However, we have found, using initial data (500 mb.) for Hurricane Ione, September, 1955 obtained from the U. S. Weather Bureau, that the hurricane does not get lost in the grid mesh, although the calculated closed contour lines around the hurricane do not remain circular. In the point vortex method described in §4, the vortex is necessarily circular. Possibly, with better initial data in the neighborhood of the hurricane a more nearly circular shape might be retained, assuming of course that the actual hurricane contour lines are approximately circular.

Partial success was attained in predicting the path of Hurricane Ione over a 24 hour period, even though the boundary data was held fixed in order to simplify the coding. The direction of motion was simulated and the final position (+24 hours) which was more than 300 miles northwest of the initial position was in error by 50-100 miles. The non-circularity of the calculated contours does not allow a better estimate of position. Although the 24 hour position was in our opinion well-predicted considering the possible inaccuracies of initial and boundary data, the intermediate hourly time-steps show that the calculated motion took a large "jump" in the first time-step and the motion there-

after was extremely slow. It is not clear at present whether this behavior is due to improper initial or boundary data or both and future tests are being set up to determine the causes. We are more inclined to place the blame on improper initial data. Hence the following smoothing procedure is now being introduced into the code:

1) From the initial data which we call ψ, we calculate the distribution $\tilde{F}(x,y) = (\Delta - \kappa^2)\tilde{\psi}$ at each grid point.

2) Using this $\tilde{F}(x,y)$ we solve for the smoothed initial data which we call $\bar{\psi}$ by integration, $(\Delta - \kappa^2)\bar{\psi} = \tilde{F}$. We expect this smoothing procedure to eliminate the abrupt jump in the numerical solution. Also time-dependent boundary data will be used henceforth.

An indication of the rather tentative nature of our test runs to date is illustrated by our uncertainty concerning a good estimate of the parameter κ. Our tests so far have indicated an apparent insensitivity of the numerical calculations to values of κ differing by a factor of two or three. This insensitivity seems very questionable.

4. NUMERICAL SOLUTION BY THE POINT VORTEX METHOD

In the point vortex method we carry out a separation (see equation 3a) of the symmetrical hurricane field ψ_o from the remaining regular field ψ_1 from the same initial data (500 mb) used in §3 for Hurricane Ione, September 1955. Since we have not been able to obtain reliable local velocity distributions in the neighborhood of the hurricane eye, we have encountered difficulty in getting a good estimate of the hurricane strength γ (see equation 2a). One method which we have used to obtain γ is by extrapolation of the distribution of the quantity $r(\psi_o)_r$ to the origin,

$$\frac{\gamma}{2\pi} = \lim_{r \to 0} r(\psi_o)_r \qquad (7)$$

where r is the radial distance from the eye. However, none of the available data gives us much confidence in the calculated values of γ. A general criterion which we have used in estimating γ is to pick the strength so that the regular field $\psi_1 = \psi - \psi_o$ has smooth, nominally-parallel contours at and near the eye with spacing and direction approximating the estimated initial velocity of the hurricane eye. Even if reliable data were available, precaution must be taken in separating ψ_o from the total ψ in order to avoid unrealistic residual kinks in the contours of ψ_1 at and near the vortex point (x_o, y_o). In the first test run which we made by the point vortex method, we apparently encountered such difficulties. Unfortunately, the coding for this particular run was set up to yield the time-wise vortex position

(x_0,y_0) only and not the spacial ψ_1 distribution so that we do not have complete confirmation of our difficulties. This will be remedied in future test runs. In addition, both the smoothing procedure for the initial data (for ψ_1) and time-dependent boundary data will be used in the future (cf. §3 for total ψ). An outline of the numerical method of solution by the point vortex method is described in IMM-NYU 247 (1958).

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